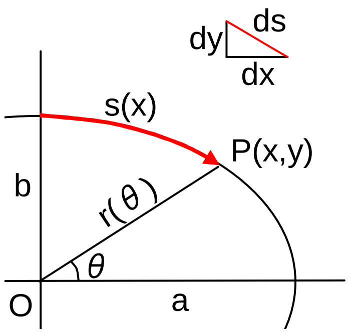
How to find elliptic arc length

2012. 9.20 keisan



elliptic equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad y=\frac{b}{a}\sqrt{a^2-x^2}$$

arc length s(x)

$$s(x)=\int_0^x \frac{ds}{dx}dx$$

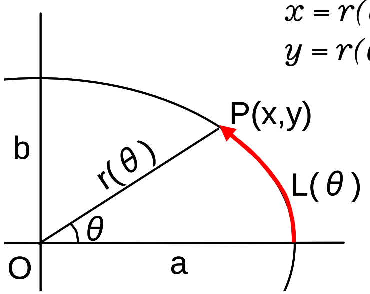
$$=\int_0^x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \qquad ds^2=dx^2+dy^2$$

$$=\int_0^x \sqrt{1+\frac{b^2x^2}{a^2(a^2-x^2)}}\,dx \qquad \frac{dy}{dx}=\frac{-bx}{a\sqrt{a^2-x^2}}$$

$$=\int_0^x \frac{\sqrt{1-k^2\left(\frac{x}{a}\right)^2}}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\,dx \qquad k^2=1-\frac{b^2}{a^2}$$

$$=a\int_0^{\frac{x}{a}} \frac{\sqrt{1-k^2t^2}}{\sqrt{1-t^2}}\,dt \qquad t=\frac{x}{a}$$

$$=aE\left(\frac{x}{a},\,k\right) \qquad E(x,k)=\int_0^x \frac{\sqrt{1-k^2x^2}}{\sqrt{1-x^2}}\,dx$$



$$x=r(\theta)\cos\theta$$
$$y=r(\theta)\sin\theta$$

arc length L(θ)

$$L(\theta)=s(a)-s(x)$$

$$=a\left\{E(k)-E\left(\frac{r(\theta)\cos\theta}{a},\,k\right)\right\}$$

$$r(\theta)^2=\frac{a^2b^2}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$s(x)=aE\left(\frac{x}{a},\,k\right)$$